Simple Beautiful

$E=mc^2$

$E=nh\nu$

$L=n\hbar$

THE BOOK POINT

Lambodar Prasad Singh

Simple Is Beautiful

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SIMPLE IS BEAUTIFUL

by **Lambodar Prasad Singh**

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SIMPLE IS BEAUTIFUL
Twelve Equations and the New World-Vision

"An equation for me has no meaning unless it expresses a thought of God."

S. Ramanujan

Nature! Out of the simplest matter it creates most diverse things, without the slightest effort, with the greatest perfection, and on everything it casts sort of fine veil. Each of its creations has its own essence, each phenomenon has separate concept, but everything is a single whole.

-Göthe

Dedicated to the memory of Professor L. O' Raifeartaigh whose inspiring scholarly and humane influence in Dublin Institute for Advanced Studies opened me, albeit in a very small way, to the process of understanding the deeper meaning of fundamental physics during the two years of my study and research there.

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PREFACE

Nature is, amazingly enough, fundamentally simple. This simplicity hidden behind the 'fine veil' is reflected in the simple mathematical equations which describe it. The profound physical insights underlying these simple equations are indeed beautiful and have played key roles in fashioning our understanding of the Nature.

I came to realize these in course of my teaching of Quantum Mechanics. I shared my excitement with students of every passing batch. Soon it occurred to me to let loose my excitement beyond the four walls of lecture room-to make my excitement trigger, hopefully, the same in young minds out there. Hence this attempt.

The measure of appreciation of the essence-simple is beautiful-this writing succeeds in creating in the minds of the readers, to that only this effort would be considered worth-while and rewarding.

Lambodar Prasad Singh

Simple is Beautiful

Simplicity has a beauty of its own. A beauty that has enchanted the rich and creative minds ages over. Such minds, immersed in their world of simple beauty, have infact given shape and direction to human civilization through their intellectual constructs. These constructs, in turn, constitute the world of knowledge -the real world heritage.

"Simple living and high thinking" is not merely an empty calling. Simple living is an essential prerequisite to high thinking. The saints and sages of the past and the thinkers of today as well, all live a life of great simplicity with barest of matter necessary to support life. Spirit grows in the space emptied of matter. Renunciation of desires to possess is the hallmark of great characters. Lives of Budha, Socrates, Confucius, Christ down to Ramakrishna, Vivekananda, Ramana Maharshi, Gandhi, Tolstoy and Tagore are all living testimonies to the miracle of simple living. They all enchanted the public minds of their time by serene beauty of their lives. It is their

"footprints on the sands of time" that the world cherishes to follow .

Simple living has the potentiality of engendering high thoughts expressed in simple words. Christ said; "hate the sin not the sinner". His immortal uttering of forgiveness towards his executors -"God forgive them, for they know not what they are doing"- will continue to inspire righteous people to fight the evil braving greatest of humiliation and torture. Deeply distressed by the sufferings of human life, Budha proclaimed his revelation to the world in simple words; "death of desire is death of the sufferings." Tagore's call "if no one responds to your call, walk alone" will always be the solace of one who follows conscience. Gandhi never wavered from truth, nonviolence and his conviction that "end can not justify the means". Thoughts such as these continue to guide the humans and their civilization over ages.

Perhaps the simplest and most profound statement of philosophy was expressed in the words "SOHAM". Just as two words combine to create this single word, this simple single-word sentence connected, in turn, every part of the creation to the whole of it in an organic manner. The mathematical rendering of this expression could be an equation like

He=me. A thought of unity of pristine purity and fragrance. Who indeed can escape its charm?

Science too, as an integral component of human creation, has also witnessed great revelations of Nature's architecture, design and function in simple equations. The simple mathematical equations of Newton's laws of motion and gravity raised to greater perfection in the hands of Lagrange and Hamilton, of Maxwell's electromagnetic phenomena, of Boltzmann's statistical mechanics and of Huygen's intuitive construction, form the core of mechanics, electromagnetism, thermal physics and optics, which combine to create the entire edifice of classical physics. Classical physics, however, basically is the physics of macro systems. As man found means of gleaning deeper-delving into the world of micro-the equations began to look even simpler. This is, as it were, to drive home the point that Nature at its fundamental level loves simplicity.

Look at the chemistry of matter. Large complexity of myriad forms of matter arises out of chemical aggregation of large number of atoms of just ninety two elements. This is a simplification of invaluable significance for human comprehension of the seemingly incomprehensible world.

When physics dug deeper to unravel the fundamental laws, a large number of simple equations emerged. Max Planck's equation of quantization of energy ($E = nh\nu$), Niles Bohr's equation of quantization of angular moment (L=n$\hbar$), Louis de Broglies' equation for wavelength of probability waves of matter ($\lambda = \frac{h}{p}$), Erwin's Schrödinger's equation of wave-mechanics $H\psi = E\psi$ and finally Werner Heisenberg's uncertainly relation ($\Delta x.\Delta p \sim \hbar$) are all stunningly and deceptively simple. (In the above equations E is energy, h is Planck's Constant, n is an integer, ψ is a complex-valued function, ν is frequency, λ is wavelength, p is momentum, Δx and Δp are small uncertainties in measurement of position and momentum). Each of them reveals profound nuances of Nature at very fundamental level. Not surprisingly, therefore, each of them has earned Nobel prizes in Physics for their respective creators. Such equations build the structure of quantum mechanics, which has been rightly termed as one of the two most important creations of human mind in twentieth century along with theories of relativity. Quantum mechanics, indeed, is the language of Nature.

It is said, "loveliness never dies, it passes on to other loveliness". Rabindranath Tagore captures this transcendence of sweet soft loveliness in exquisite

words; "yes, when the mother was a young girl it lay pervading her heart in tender and silent mystery of love- the sweet, soft freshness that has bloomed on baby's limbs." Perhaps, this all-pervading universality of beauty, from the innermost to the manifest, applies to all aspects of creation. Be it in philosophy, be it in religion, be it in literature, be it in human mind or be it in cosmic order, simplicity begets greater simplicity. Beauty begets greater beauty. And both, perhaps, are synonymous with truth that human mind so passionately seeks. As John Keats so eloquently puts it, "Beauty is the truth, truth beauty. That is all ye know on earth and ye need to know". Yes, simple is beautiful indeed; as beautiful as beautiful can be !

1858-1947

Max Karl Ernst Ludwig Planck was born in Kiel, Germany in 1858. Most of his youth and initial University years were spent in Bavaria, South Germany. Despite early interest in music and humanities, he was attracted most to mathematics and theoretical physics. He went to Bern in 1878 to study with professors Hermann von Helmoltz and Gustav Kirchoff. He earned his doctoral degree at the University of Munich in 1879. He had his early appointments in Munich and Kiel. Then he moved to prestigious Physikalische Techniche Reichsanstalt, Berlin in 1889. He remained there till 1928. Here he invented his quantum theory.

During Hitler's Nazi regime 1933-45, though opposed to it as President of Kaiser Wilhem Society, he did not leave Germany as many of his famous scientist friends did. He died on October 3, 1947.

PLANCK AND HIS MAGIC EQUATION

Physics witnessed a revolution of sorts at the turn of the last century. Towards the last decades of 19^{th} century, physicists were grappling with the problem of understanding the distribution dependence of energy of black-body radiation with the wavelengths present in it; the so-called famous black-body radiation spectrum. With no solution available within contemporary knowledge of the time, Max Planck, more out of desperation than of logical compulsion, introduced the concept of quantization of the Black-body radiation energy in the year 1900. This, to every one's amazement, provided a proper understanding of the black-body radiation spectrum and gave birth to Quantum Mechanics.

The complete mathematical formulation of this new mechanics took nearly the first quarter of twentieth century. When complete, quantum mechanics brought in a number of wonderful new concepts with regard to understanding the physical

world at the fundamental level. Grossly speaking, quantum mechanics is the mechanics of the "small", the small meaning as small as an atom or smaller. Such small things actually constitute the physical world at the fundamental level. Quantum mechanics, so it is said, is the language of the Nature.

In course of the twenty five years taken to complete the mathematical formulation of the Quantum Mechanics, a number of simple equations were written which were not just mathematically beautiful but represented powerful thoughts, which shaped our understanding of the physical world around us.

Let us start with Planck's

$E = nh\nu$,

where E is energy of radiation with frequency ν, n is an integer and h is a constant which is today called Planck's constant after the name of its creater. This equation tells us that the energy of radiation of frequency ν can only be an integral multiple of the smallest energy quantum given by the product of h with the frequency ν. Thus, $h\nu$ can be taken as the "atom" of radiation of frequency ν.

Prior to Planck's proposition of such quantization of energy, it was assumed that radiation possesses continuous energy values. By introducing the revolutionary idea of quatisation of energy-albeit in

terms of tiny units-Planck could derive the exact distribution of energy of black-body radiation in terms of its wavelength as obtained in the experiments. The hidden characteristic of Nature in the form of energy quantization was finally laid bare!

The reason as to why the knowledge of such quantization of energy was not obtained earlier can be easily understood by recognizing that quantum of energy $h\nu$, indeed, is an incredibly small amount magnitude wise. It is this smallness of the quantum that made the energy of radiation look continuous until technology developed to measure such small amount of energy or man's mind could grow to deal with physical systems where such quantization of energy could play a significant role.

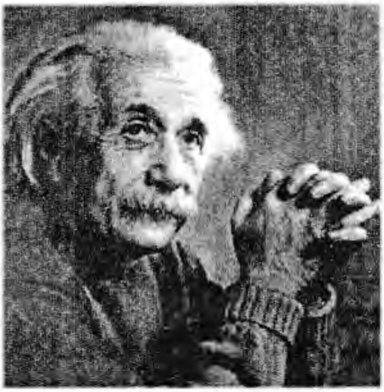
Max Planck was awarded the Nobel prize in physics in the year 1918 for introducing concept of quantization of energy and laying the foundation of the beautiful structure of quantum mechanics.

Is not Planck's equation both simple and beautifully revolutionary!

The outside world is something independent from man, something absolute and the quest for laws, which apply to this absolute, appeared to me as the most sublime scientific pursuit in life.

-MAX PLANCK

ALBERT EINSTEIN



1879-1955

Albert Einstein was born in Ulm, Germany on March 14, 1879. The Einsteins moved to Munich when Albert was one.

He finished his secondary education at Gymnasium in Arau, Switzerland and continued his studies at the renowned Eidgenossische Technische Hochsuhule in Zurich. Later he worked as technical expert in the patent office in Bern from 1902 till 1909. During this period, the happiest of his life, he married Mileva Maric, had two sons Hans and Eduard and wrote three monumental papers on Special Theory Relativity, Brownian Motion and Photoelectric Effect-all completed in 1905. He occupied Professor position at the University of Zurich, Karl-Ferdinand University in Prague, Eidgenossische Technische Hochschule in

Zurich. He joined the University of Berlin as Professor in 1914 and wrote his papers on General Theory of Relativity there which catapulted him to become one of the most famous scientists of the world of all times. He joined as Professor in the Institute of Advanced Study, Princeton, USA in 1932.

It may be mentioned that Einstein divorced his wife Mileva in 1919 and married his divorced cousin Elsa Einstein Lowenthal in the same year. However, his divorce paper stipulated that Mileva would receive, in due course, Einstein's Nobel Prize money. In 1923 the entire US $ 32,000 of Einstein's 1922 Nobel Prize money was indeed transmitted to her.

Not happy to stay in Germany after Adolf Hitler became Führer in 1933, he took up the American citizenship. He declined the offer of Israeli presidency in 1952 following the death of Chaim Weizmann.

Einstein died at Princeton, USA on April 18, 1955.

PHOTOELECTRIC EFFECT AND EINSTEIN

We have already discussed about the Plank's magic equation, $E = nh\nu$. This equation, as we said, ushered in the age of quantum mechanics. The first application of Planck's equation, after Planck himself, was made by Albert Einstein in providing a correct understanding of photoelectric effect.

Photoelectric effect, discovered in 1900 by Lenard and others, as the name suggests, refers to ejection of electrons (electric effect) from the surface of a metal plate subjected to an energetic beam of light (photos). Important characteristics of this effect are:

a. The electrons come out only when the frequency of light exceeds a certain value and once the required frequency is attained ejection is nearly instantaneous.

b. The energy of ejected electrons depend on the frequency of incident light and not on its intensity.

c. The number of electrons ejected i.e. the strength of electric current produced depends on the intensity of incident light.

All these characteristics, once again, were difficult to understand on the basis of classical electromagnetic wave theory. The classical electromagnetic theory when applied to such a process, predicts that ejection of electrons from metal plate would take nearly a few seconds, sometimes minutes, or even hours instead of being instantaneous. Further, it was expected that the electron energy would depend on intensity of incident light. The observed dependence on frequency of incident light was a mystery.

Einstein, however, quickly realized that the answer to the understanding of photoelectric effect lies in the application of Planck's equation by postulating, that light, not only in radiated form as potulated by Planck, but in all forms of its existence has quantized energy. So, the incident light energy is $h\nu$. The energy of electrons ejected in photoelectric effect is then given by Einstein's equation:

$$E = h\nu - \phi .$$

This equation was written down in the year 1905. Here E is energy of ejected electrons, $h\nu$ energy of the photon of incident light and ϕ the work function or the minimum amount of energy required for

electrons to be emitted. This may be viewed as the collective energy of attraction experienced by the electron due to the ions present in the metal.

From the above equation, it is clearly seen that energy of electrons is indeed dependent on frequency of incident light and not on its intensity. This equation also indicates that ejection of electrons from the metal plate is instantaneous once the energy of the incident light exceeds the value of work function, which depends on the nature of the metal plate used. The number of ejected electron, of course, now depends on the intensity of light or the number of photons present in the beam. Thus all experimental facts stand explained.

Einstein was awarded the Nobel Prize in Physics in 1921 for providing the theoretical understanding of the photoelectric effect just five years after Planck's quatisation of energy of black-body radiation was proposed. The main import of Einstein's work was introduction of the idea of photon and viewing light as collection of immensely large number of them.

Is not Einstein's equation both simple and deeply beautiful?

The most beautiful thing we can experience is the mysterious. He to whom this emotion is a stranger, who can no longer wonder and stand rapt in awe, is as good as dead: his eyes are closed.

-EINSTEIN

NIELS HENRIK DAVID BOHR

1885-1962

Niels Henrik David Bohr was born in Copenhagen, Denmark on October 7, 1885. Bohr matriculated from the University of Copenhagen in 1903. He earned his Master of Science degree in 1909 and doctorate in 1911. He then worked with Sir Joseph John Thomson at Cavendish Laboratory, England and with Ernest Rutherford at Manchester, England.

After stints of teaching assignments at Copenhagen University (1913-14) and Victory University (1914-16), he became a Professor of theoretical physics at Copenhagen University in 1916. He then became the Director of the Institute of for Theoretical physics (later to be called Niels Bohr Institute) created for him by the University in 1920 and remained so till his death.

He provided the complimentarity principle to the mutual exclusiveness of "classical macro-world" and "quantum microuniverse" in terms of pure philosophy with an example provided by Sophocles', "Antigone". Here the mutually exclusive points of "social duty" and "commitment to family" was resolved as complementary nonetheless. Such are the ways of the atom, Bohr posited.

Bohr died in Copenhagen on November 18, 1962.

He was married to Margarethe Norlund and had six sons. One of them, Aage Bohr, was awarded the Nobel Prize in Physics in 1975.

BOHR'S ATOM

We have seen in our last two discussions that through the successes of the equations written by Planck and Einstein in understanding black-body radiation spectrum and photoelectric effect respectively the idea of quantization of energy as one of the fundamental property of energy was established.

Then came the amazing experimental result of Rutherford and his collaborators Geiger and Marsden in the year 1909 that the atom, thought be an indestructible charge-neutral constituent of matter, has a structure. In the middle of an atom, there is a positively charged massive nucleus, carrying almost the total mass of an atom. Around the nucleus, circle a number of nearly massless negatively charged electrons. The number of circling electrons is so fixed that the total -ve charge they carry exactly cancels the +ve charge of nucleus to make the atom electrically neutral.

So far so good. But the most perplexing question that this experimental result raises is about the stability of the atom. Since opposite charges are

known to attract each other, the attraction between electrons and nucleus should make the electrons collapse onto the nucleus in course of time thereby making the atoms unstable. With unstable atoms, evidently, no matter could be there in stable form in the Universe. Nor will we be.

Here enters Niels Bohr with his amazing postulates in the year 1913. Much in the vein of quantization of energy, Bohr postulated that the angular momentum of electrons rotating around the nucleus of the hydrogen atom is quantized! Thus, out of all possible orbits available for the revolving electron, only those are allowed for which

$$L = n\hbar$$

Here L is magnitude of angular momentum of the electron, n is an integer and $\hbar$ is the Planck's constant h divided by 2π. So $\hbar$ is the minimum value of angular momentum just as $h\nu$ is the minimum value of energy for radiation of frequency ν. While stationed in the orbit the electrons do not radiate energy. Clearly, the postulate of Bohr prevents the catastrophic collapse of electron onto the nucleus ensuring stability of the atom.

Using Bohr's postulate the energy of the revolving electron in various allowed orbits can be calculated. If electron jumps from a higher energy level to a lower

one, the difference in energy between two levels (ΔE) is radiated out. The frequency of this radiation should be such that $\Delta E = h\nu$. This provided the basis to understand the complicated spectra of radiation emitted by the atoms which infact was known much earlier and was waiting for a theoretical explanation.

Nobel Prize in physics for the year 1922 was awarded to Niels Bohr for providing this understanding of the atomic structure, its stability and the spectra.

Is not Bohr's equation both simple and profoundly beautiful in its physical contents?

A shallow truth is a statement whose opposite is false; a deep truth is a statement whose opposite is also a deep truth.

NIELS BOHR

Louis Victor de Broglie

1892-1987

Louis-Victor-Pierre-Ramond Prince de Broglie was born on August 15, 1892 at Dieppe, France. The de Broglies were a distinguished French family who had served in high government and military positions for generations. The title of Duc de Broglie was granted in 1742 to the eldest living son, all other sons having the title "Prince".

Louis de Broglie obtained his degree from Sorbonne in 1909 in the subject of medieval history. Under the influence of his teacher Paul Langevin and having had the opportunity of going through the proceeding of First Solvay Congress of 1911 held in Brussels which was getting edited by his elder brother Maurice and Langevin, he vowed to devote all his "efforts to achieve an understanding of the mysterious quanta". He then, obtained his degree in science and

doctorate from the University of Paris in 1913 and 1924 respectively. He joined teaching assignments at Sorbonne and at Institute Henri Poincare. In 1932 he became Professor of theoretical Physics at University of Paris.

Louis de Broglie received the first Kalinga award in 1952 for his efforts to explain intricacies of modern physics to laymen.

He died on March 19, 1987 in Louveciennes, France.

DE-BROGLIE'S PROBABILITY WAVE

We have discussed earlier how Einstein explained the photo-electric effect by postulating that quantization of energy is a fundamental property of radiation in all its form. We also saw that Niels Bohr laid the foundation of understanding the atomic spectra by providing a simple model of the hydrogen atom through his postulate of quantization of angular momentum.

The next fundamental contribution in the development of quantum mechanics was given by Louis de Broglie. It rarely happens that a person of royal lineage ever gives himself to the study of a serious subject like Physics let alone making a revolutionary contribution to it.

Louis de Broglie took the cue from Planck and Einstein. He argued that if radiation possesses both wave and particle (photon) characteristics, why should the same be not true of the particles? Driven by such an argument, he postulated that particles too

should be associated with a wave characteristics. The wavelength (λ) should be given by the equation:

$$\lambda = \frac{h}{p}$$

where h is Planck's constant and p is the momentum of the particle. The wave nature of electron was experimentally confirmed by Davission and Germer in 1927.

This wavelength, however, does not pertain to the real 3-dimensional waves like those of water or sound. Instead, this wavelength represents probability wave. The square of the amplitude of this wave is interpreted to represent the probability of finding the particle at a given position. Because of the dominance of ideas of wave and probability in its formulation, quantum mechanics is also often described as wave mechanics or probabilistic mechanics.

It must be emphasized that de Broglie's equation completes the notion of assigning dual characteristics to both radiation and particle. The wave and particle characteristics viewed as opposite ones in classical theory become complimentary to each other in quantum theory which, let us remind ourselves, assumes significance for systems of atomic dimensions only.

Louis de Broglie was awarded Nobel Prize in Physics for the year 1929.

Is not Louis de Broglie's equation, once again, both simple and beautiful?

In recent years, I have been led to regard the concept of complementarity with increasing suspicion.

LOUIS DE BROGLIE

Erwin Schrödinger

1887-1961

Erwin Schrödinger was born on August 12, 1887 in Vienna, Austria. He did not attend school till he entered Gymnasium in 1898, which he passed in 1906 placed first among all successful candidates. Though interested in all the subjects of humanistic curriculum, he formally studied physics under Friedrich Hasenohrl, a student of Ludwig Boltzmann. He completed his doctorate in 1910 and started working with Franz Exner, another of Boltzmann's pupils.

He became a full professor at the University of Zurich in 1920. Moved to Berlin as Max Planck's successor in 1927. On Adolf Hitler's secession to power he left for United States and then to England. He returned to Austria to occupy a chair at the University of Graz in 1936. In 1938 with "Anschluss",

he went to Italy and finally occupied the directorship at Dublin Institute for Advanced Studies where he remained till 1956 before returning to Austria.

He was a "Polymath". He was one of the few scientists in twentieth century with agility of mind and depth of knowledge to achieve distinction in many other fields besides physics. He was a philosopher, a biologist, a historian and a scholar of literature and languages.

He married Annemarie Bertel on April 6, 1920. Childless, Schrödinger's marriage was not without its crises but both managed to stay together. Annemarie was his most devoted companion.

Schrödinger died in Vienna on January 4, 1961 and was buried in his beloved Tirolean village of Alpbach.

SCHROEDINGER'S EQUATION

Inspired by De Broglie's work of associating a wavelength with a particle, Schrödinger thought of writing an equation to describe the single particle dynamics involving the wave associated with the particle. This equation will be the wave analog of particle equations

Here H is the energy operator called Hamiltonian, E is energy and ψ is called the wave function. In general, ψ can be complex function but E being a measurable quantity is always real.

An equation of above type where an operator (here H) acting on a function (here ψ) gives the same function multiplied by a real number (here E) is called eigen-value equation. Here E is the eigen value and ψ is the eigen function. The German word "eigen" means "one's own". Which means Hamiltonian operator's own-value is E and its own function is ψ .

The physics interpretation is that ψ is an eigenstate of Hamiltonian operator with energy E. In general a complex function of coordinates, ψ was interpreted by Max Born in 1926 as probability

amplitude or the amplitude of the de Broglie's probability wave associated with a particle. The modulus square, $|\psi|^2$, is the probability of finding the particle at a definite position carrying energy E.

However, a general function ψ may not be an eigenstate with respect to an operator, say, Hamiltonian. Then ψ could be written as a linear combination of all possible eigenstates like

$$\psi = \sum_n a_n \ \phi_n$$

where ϕ_n stands for number of possible eigenstates, $a_n s$ are complex coefficients whose modulus square, $|a_n|^2$, give the probability of the physical system being observed in eigenstate ϕ_n with energy E_n. Obviously, since the total probability of finding the physical system in all possible eigenstates is unity, $\sum_n |a_n|^2 = 1$

The remarkable thing about Schrödinger's wave mechanics as described above is that it is a probabilistic mechanics as opposed to classical mechanics which is a deterministic mechanics. This shift from absolute determination of physical observables to acquiring only a probabilistic information of these values at the best, is not a consequence of our lack of proper understanding of the system. It is rather an intrinsic characteristic of

the micro-system obeying the rules of quantum mechanics.

Erwin Schrödinger was awarded the Nobel Prize for Physics for his wave-mechanical formulation of Quantum Mechanics in the year 1933.

Once again Schrödinger's equation demonstrates great beauty of fundamental physics through its simplicity.

A moderately satisfying picture of this world has only been reached at the high price of taking ourselves out of the picture, stepping back into the role of a nonconccerned observer.

SCHROEDINGER

WERNER KARL HEISENBERG

1901-1976

Werner Karl Heisenberg was born in Wurzburg, Germany on December 5, 1901. He attended the Maximilian Gymnasium in Munich until 1920 and then entered the University of Munich to study theoretical physics with Arnold Sommerfeld. He earned his doctorate in 1923 from the University of Munich. Thereafter, he joined the University of Göttingen, then known as the centre of the world mathematics, as Max Born's assistant. He became a Professor of theoretical Physics at the University of Leipzig in 1927. He then moved to Berlin as a Professor of Physics in 1942 and was appointed Director of Kaiser Wilhelm Institute, the highest research position in Germany.

He worked on nuclear fission for the war project during IInd World war, not supporting German

Government's goal of building an atom bomb but regarding the project as presenting a possibility of producing energy from Uranium. After the war, he and other physicists from the Institute were imprisoned in England from May 1945 to January 1946.

From 1946 to 1958, Heisenberg worked for restoring sciences and supporting rising young scientists in Germany. In 1957, he joined a group of physicists known as "the Göttingen Eighteen" to speak out against the nuclear bomb.

He wed Elisabeth Schumacher on April 23, 1932 and had seven children.

He was an excellent pianist. Found great pleasure in playing piano, reading philosophy and hiking in mountains.

Heisenberg died in Munich, Germany on 1st February, 1976.

UNCERTAINTY PRINCIPLE OF HEISENBERG

Yes, it may sound as contradiction of terms. One could argue that a principle always needs to connote certainty; how can a principle talk of uncertainty? The truth of the matter, however, is that the principle speaks of the certainty of the uncertainty.

As we discussed earlier, Erwin Schrödinger in 1926 created wave mechanics working with differential equations and continuous wave functions. Schrödinger claimed that nature exhibited no quantum jumps at all. Heisenberg in 1927 contradicted this and derived a remarkable relation

$$\Delta p \, \Delta q \sim \hbar \, .$$

Here Δp and Δq are uncertainties in the measurement of momentum p and position q respectively and h is the Planck's constant. Since h, though extremely small, is not exactly zero, this relation clearly suggests that if either momentum or position is measured with infinite accuracy the other is infinitely inaccurate in a simultaneous

measurement. This inaccuracy is not a consequence of any deficiency of either the process of measurement or the measuring instrument or even of the measurer; it is the intrinsic characteristic of a micro system-the physical system described by the rules of quantum mechanics, or in other words by the Schrödinger's wave mechanics. Thus, Heisenberg's uncertainty principle in simultaneous measurements of momentum and coordinate is a law of Nature.

Heisenberg was awarded Nobel Prize in Physics in 1932 for "creation of quantum mechanics" of which uncertainty relation forms an unshakeable foundation.

Actually, any attempt to measure the position of a system like an electron which obeys rules of quantum mechanics, the radiation used in the experiment to see the electron also changes its momentum Thus, the measuring "instrument" including the observer become an integral part of the measuring process. The result of measurement, thus, becomes observer-dependent. So the observed and the observer from an integral whole. Therefore, "the reality is partly observer created". Reality, in other words, is not reality unless it is an observed reality.

This observer-dependent reality invalidates the basic dictum that physics and, for that matter, whole

of science hitherto held closest to its chest for centuries, i.e. the world has an objective existence. Quantum mechanics proclaims that the world, at the fundamental level, has no objective existence. Just as Schrödinger's work overthrew the Newtonian view that the world runs in a clock-like fashion as per predetermined rule giving way to a probabilistic world-view, Heisenberg's equation discarded absolute objective existence of the physical reality.

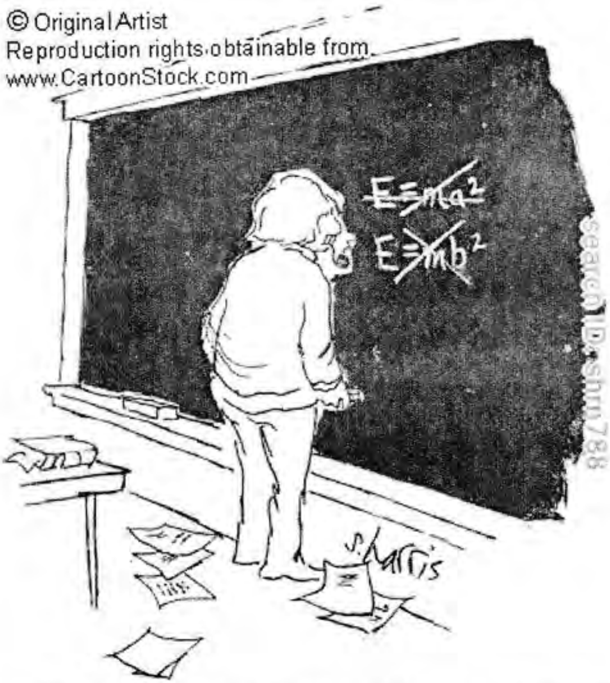
The revolutionary change in both scientific and philosophical way of looking at the world ushered in by Quantum Mechanics, can effectively be ignored when one is considering macro systems like, say, the motion of a football instead of an electron. Magic of quantum mechanics is at its manifest best when one studies microsystems whose dimensions are nearly that of an atom (10^{-8} cms) or smaller.

The simple-looking Heisenberg uncertainty relation is not only simple but also profoundly revolutionary!

Beauty is the proper conformity of parts to one another and to the whole.

-HEISENBERG

UNITY OF ENERGY AND MASS



$E = mc^2$ happens to be the most famous and popular of all physics equations. Even people with little acquaintance with physics or even science seem to have heard about it. The celebrity status of the equation is mainly the reflection of that of its inventor, Albert Einstein, one of the most famous of physicist of all times. He arrived at this equation as a sequel to his formulation of special theory of relativity in 1905.

Principles of conservation of energy and mass held very high place in pre-relativity days. The principle of conservation of energy, first advanced by Leibnitz as early as seventeenth century, was further developed in nineteenth century.

Consider the swinging motion of pendulum. When the mass is at the highest point it is at rest and as it comes to the lowest point the height disappears but acquires the celocity(v) as if the height is converted into velocity. Infact this is expressed through the equation $mgh = 1/2mv^2$. Here mgh, the product of mv^2 mass(m), acceleration due to gravity(g) and heigh(h) above the lowest point of motion stands for potential energy and $1/2mv^2$ stands for kinetic energy at the lowest point. So, here we find that energy is conserved for the pendulum though it changes from potential form to kinetic form as the mass point changes its position from highest to the lowest one.

Now, if the pendulum stops due to friction, it has been established that mechanical energy is converted to heat. Thus, conservation of mechanical and thermal energy were merged into one. With time, the conservation principle was extended to chemical and electromagnetic processes. So, the sum total of energies remained constant through all changes that might occur.

The principle of conservation of mass simply means that mass remains unchanged under any physical or chemical changes like heating, melting, vaporization or combination into compounds. This principle, however, proved inadequate in the face of special theory of relativity where mass was shown to vary with velocity. Velocity being related to kinetic energy it therefore naturally got merged with the energy conservation principle through Einstein's equation

$$E = mc^2$$

Here c=186000 miles per second, is the velocity of light. So the energy conservation principle swallowed within itself the principle of mass conservation just as it did with conservation of heat earlier.

The equation $E = mc^2$ means that a small amount of mass is equal to tremendous amount of energy as square of a large number like c has to be multiplied to mass to obtain its energy equivalent. This went unsuspected for so long since processes where such conversion of mass into energy occurs was not known until radioactive disintegration of atoms was observed. In the radioactive disintegration processes the validity of $E = mc^2$ is verified.

Einstein's equation, as it so happens, also provides the theoretical basis of atomic weapons. Averting

threats of use of such weapons has become the most urgent problem of our time.

In the end, what we find is that mass can be loosely considered as condensed energy and energy as diluted mass. A great unification of mass and energy! Once again, we come across a simple equation but immensely profound.

To repeat, simple is beautiful indeed.

Subtle is the Lord, malicious He is not.

-EINSTEIN

SPACE AND TIME TO SPACE-TIME

Ingenious experiments conducted by Oersted, Ampere and Faraday during nineteenth century revealed that electric and magnetic effects are not independent phenomena as taken for granted over centuries. It could be demonstrated that electric currents create magnetic field and vice-versa. This was, perhaps, the first example of seeing 'unity in diversity' in Physics. The second was the unification of mass and energy in Special Theory of Relativity (STR) as discussed in the previous essay. However, the most profound of all unification fashioned through STR was that of space and time. It revolutionized our outlook about the physical world.

We all live in space. We all change with time. As such, we and our experiences are intimately entangled with space and time. Our room has extensions in three independent directions in its length, breadth and height. Stationed at a point we can move forward or backward, left or right and up or down. Such experiences lead us to take space as 3-dimensional.

Time, in the same sense, is perceived to be 1-dimensional through its monotonous flow forward. Further, as our experiences would have it, the space-with its 3-dimensional foliation and time with its 1-dimesioanl flow are distinct entities. This, too, was the scientific perception till Einstein arrived on the scene.

In a flash of inspired intuition Einstein propounded the ingenious idea that our world is 4-dimensional, where the 3-dimensional space and 1-dimensional time are integrated into a 4-dimensional space-time whole. In 3 spatial dimensions the length square of a vector $\vec{R}$ is defined as sum of the squares of its components along x, y and z axes

$$|R|^2 = R_x^2 + R_y^2 + R_z^2 = R_1^2 + R_2^2 + R_3^2$$

and is an invariant under rotation. Similarly, the length square of a vector $\overline{M}$ in 4-dimensional space-time is written as

$$|M|^2 = M_x^2 + M_y^2 + M_z^2 + M_t^2 = M_1^2 + M_2^2 + M_3^2 + M_4^2.$$

This too is an invariant under a four-dimensional "rotation" in a generalized sense. The difference here being that M_t^2 or M_4^2 is $-c^2t^2$ i.e. negative. Thus, the above invariance can also be written as

$$x'^2_1 + x'^2_2 + x'^2_3 - c^2t'^2 = x_1^2 + x_2^2 + x_3^2 - c^2t^2$$

Here prime (') refers to transformed quantities. The presence of –ve sign in the invariant quantity also has its consequences. The invariant quantity can be +ve, -ve or zero. Infact it is zero for light since light always moves with velocity c making

$x_1^2 + x_2^2 + x_3^2 = c^2t^2$. For all material particles of velocity v, $x_1^2 + x_2^2 + x_3^2 = v^2t^2$ being always less than c^2t^2, the invariant quantity is –ve.

This region of space-time is called causal or time-like region. For the invariant quantity having + ve value, one has noncausal or space –like region of space time. Light trajectory, thus, defines the boundary between time-like and space-like regions. Thus, if we move along, say, x-axis not only our position along x-axis changes but our position with respect to time axis also changes in order to respect the above equality or invariance. This is like spending a piece of space to buy a little of time and vice-versa. A trade-off between space and time is possible since they form an integrated whole.

The idea of space-time integration led to interesting conclusions that in a moving frame clock runs slow (time dilation), length of a rod decreases (length contraction) and mass of the moving object increases with speed. All these are experimentally verified. But such effects become appreciable only when the velocity of a moving object becomes close to the velocity of light in vacuum which is thirty crore meters per second. At ordinary velocities these dramatic effects are vanishingly small and hence not observable.

How simple an idea and how profoundly beautiful!

The most incomprehensible thing about the world is that it is comprehensible.

EINSTEIN

1642-1727

To many, Isaac Newton symbolizes the supreme intellect that human race has produced. Only Galileo and Einstein may join him in that high pedestal of matchless creativity.

Newton was born on Christmas Day of 1642, the year of Galileo's death to a family of farmers living in Woolsthorpe, England. He was born premature and was extremely frail and puny at birth. Who could imagine that an infant of such incredible fraility held within himself the genius to chart a new road for human civilization!

Newton exhibited his experimental genius as a boy in construction of mechanical toys like kite with a lantern, sundials, wooden clock etc.

Newton had his early education in Grantham Grammar School. He quickly rose to become the top

boy of the school. He joined Trinity College, Cambridge in June 1661. Newton earned his B.A in 1664. His mathematics teacher was Isaac Barrow, a mathematician of distinction , who later resigned from Lucasian chair to get his student Newton installed in his position.

The Great Plague broke out in 1664. Cambridge University was closed. Newton returned home. In his isolation at home, his meditation led him to the discoveries of calculus, the law of gravitation and composition of white light. In the last years he propounded the laws of motion and derived the Kepler's laws using calculus and his laws of gravitation. He, at the age of twenty six, succeeded Isaac Barrow as Lucasian Professor of Mathematics at the Cambridge University in 1669.

Skillfully coaxed by the famous astronomer and his friend Edmund Halley, Newton compiled all his results to write Philosophiae Naturalis Principia Mathematica (Mathematical Principles of Natural Philosophy). In 1686 the Principia was presented to Royal Society and was printed at Halley's expense in 1687.

Newton's deterministic world-view reigned supreme for over two centuries till the advent of twentieth. As physical systems of atomic dimensions

and light-like velocities started occupying human attention, inadequacies of Newtonian physics began to crop up. Attempts to circumvent these difficulties ushered in the new concepts of quantum mechanics and relativity. Newtonian world-view was not rejected but stood expanded and generalized.

Newton's obsession with mathematics and natural philosophy made him forget his body and with it also his teen-age sweet heart Miss Storey. He never loved any one else nor married.

Newton died in the morning hours of 20th March, 1727. One could only say, "Tis death is dead, not he".

GRAVITATION: NEWTON & EINSTEIN

Classical physics considered space, time and matter as three fundamental entities in the description of the physical world. Of the demise of distinctness of space and time and erection of edifice of space-time in the formulation of special theory of relativity by Einstein, we discussed earlier. Special Theory of Relativity succeeded in describing electrodynamics and mechanics in a Lorentz covariant form. Just as noninvanance of Maxwell's equations of electrodynamics under Galilean transformation led Einstein to formulate Special Theory of Relativity, here again he found that Newton's equation for gravitational interaction is not invariant under the

Lorentz transformation. This set his thoughts rolling for formulation of a relativistic theory of gravitation.

Based on Galileo's observation that the rate of fall of the bodies under free-fall is independent of their mass, Newton used his second law to conclude that the gravitational force is proportional to the mass of the body on which it acts and his third law to conclude that the force is also proportional to mass of its source. Newton in 1864 further proved that planets moving under the influence of inverse square law satisfy all the empirical laws of Kepler. Thus emerged the famous Newton's law of gravitational force between two masses m_1 and m_2 at a mutual distance r as $F = G\, m_1 m_2 / r^2$ first published in 1686. G, today, is known as Newton's gravitational constant, which carries an extremely small numerical value making gravitational force between two normal masses separated by normal distance extremely small. Gravitation, however, is universal; whenever there is mass or energy(since energy is equivalent to some mass as per equation $E = mc^2$) there is gravitation.

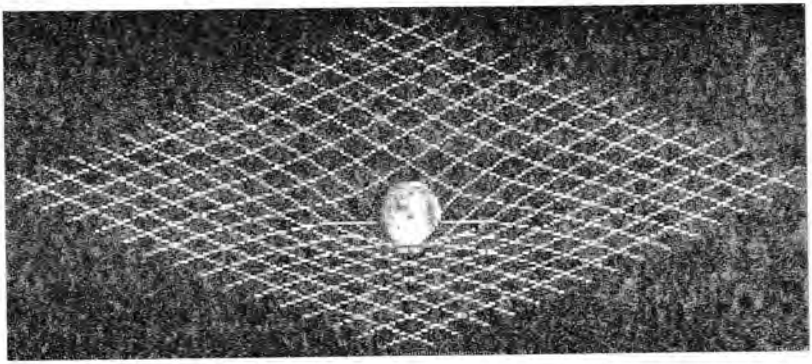
The understanding of gravitation did not change for nearly two and half centuries till Einstein appeared on the scene.

In trying to put the theory of gravitation in Lorentz covariant form Einstein generalized his Special Theory of Relativity to frames which move

with respect to each other with uniform acceleration. This generalization saw amalgamation of "space, time and matter into one fundamental and inseparable unity". If special theory of relativity pictured the physical world as a four-dimensional monotonous plane extending from infinity to infinity, the General Theory of Relativity (GTR), as a new theory of gravitation, pictured the physical world as four-dimensional space-time filled with "mole-hills which appear to us as matter". Einstein's beautiful mathematical equation written down in 1916 that represents this stupendous concept is

$$R_{\mu\nu} - 1/2\, g_{\mu\nu}\, R = -8\pi G\, T_{\mu\nu}$$

Here $R_{\mu\nu}$ and R are mathematical objects representing curvature of space-time which go to zero when space-time is flat. $T_{\mu\nu}$ in the right handside of the equation specify energy and momentum of material content of the Universe. Thus, curvature of space-time and masses present in it are integrated. The meaning of this equation can thus, be put as "mass



tells space-time how to curve and curvature of space-time tells mass how to move".

Newton's gravitational equation follows from Einstein's equation when mass giving rise to gravitational field moves with slow velocity and the gravitational field is weak and static. It correctly explained precession of perihelion of planet Mercury as it revolves round the sun. Its prediction of bending of light of a distant star passing by the Sun was confirmed in famous solar eclipse experiments of 1919. Einstein was canonized.

It is on record that Einstein's creation of GTR was inspired by what he calls "the happiest thought of his life"-namely, "an observer falling freely from the roof of a house with uniform acceleration feels no gravitational field". This idea along with Galileo's observation of matter independence of acceleration of free fall implied that relativity principle has to be extended to coordinate systems moving with uniform acceleration relative to each other.

Since we know now that gravitational force plays the key role in understanding the architecture of the Universe, it is the Einstein's equation which is used in mathematical analysis of the universe. It is this equation which tell us how was the Universe created, how it evolves and how it got the present structure of stars, planets and galaxies. It is the small fingers of

this equation, which weave the cosmic tapestry bedecked with planets, stars, galaxies and their clusters.

Einstein's General Theory of Relativity has been hailed by all who understand it as the ultimate in logical thinking. In the words of Herman Weyl, it is the "supreme example of speculative thought". Landau and Lifsitz describe it as "the most beautiful of all existing physical theories". It appeared to me, "like a great work of art, to be enjoyed and admired from a distance " says Max Born. "Scarcely any one who fully understands this theory can escape from its magic," says Einstein of his own creation. But the best, perhaps, comes through the words of Cornelius Lanczos when he writes," Theories come and theories go. Einstein did more than formulate theories. He listened with supreme devotion to the silent voices of the Universe and wrote down their message with unfailing certainty. xxx. He thus occupies a place in the history of civilization which is unique and may never be duplicated".

Einstein's simple equation, indeed, is astonishingly profound and holds the key to the beautiful architecture of the Universe!

The emotional state which leads to such achievements resembles that of the worshipper or the lover.

-EINSTEIN

1717-1783

Born in 1717 as an illegitimate child, D'Alembert was abandoned by his mother on the steps of a little chapel St. Jean le Rond by Notre-Dame in Paris to which he owes his name. The parish authorities handed the child over to the wife of a poor glazier who looked after the child as her own.

As D'Alembert grew and showed signs of genius, his real mother sent for him. But D'Alembert declined the offer and took pride in declaring the poor glazier and his wife as his parents. Throughout his life he endeavoured to keep his parents, who continued to live in their humble quarters despite their foster-son's great stature in French science, happy.

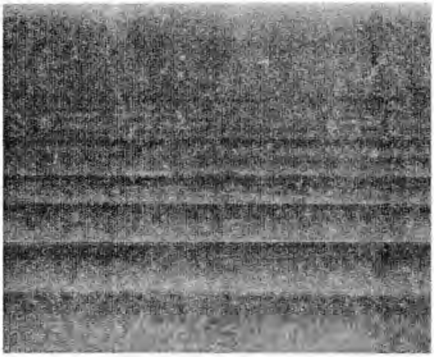
D'Alembert's most important contribution to mathematics was in partial differential equations, particularly in connection with the vibrations of

strings. It is most likely that during his preoccupation in such studies that he invented and used the wave-equation operator $\frac{1}{c^2}\frac{d^2}{dt^2}-\nabla^2$ which is known as D'Alembertian operator [¡%]. He could have been the inventer of wave equations as we find it today making its ubiquitous presence in all most all branches of physics. His other important work was enunciation of D'Alemberts principle of mechanics. This principle is regarded as the fundamental principle of mechanics.

D'Alermbert was a great friend and admirer of one of the greatest mathematicians of all times J.L.Lagrange.

D'Alembert died in 1783.

THE EQUATION OF THE WAVES



Yes, waves. We, human beings, are very much familiar with and fond of waves. As a child, we all enjoyed throwing stones into a placid pond and looking at the train of waves hitting the banks. Who has not spent hours looking at the breaking of waves at the seashore! Who has not marvelled at the running waves as the soft breeze caresses the expansive cornfields!! Not just out in space and time but within our own minds too the thoughts keep creating waves, so long as we live. It is thus but natural that wave phenomena has attracted human attention since early days. Scientific studies have revealed light and sound to be propagating in the form of waves. The story does not end here. Modern physics has also unraveled a

kind of probability wave associated with microsystems governed by the rules of quantum mechanics. Wave motion can also be associated with a periodic motion, say, that of a simple pendulum. Lo and behold; waves everywhere, in places seen and unseen, are governed by a beautiful equation.

$$\left(\sum_{i=1}^{3} \frac{d^2}{dx_i^2} - \frac{1}{v^2}\frac{d^2}{dt^2}\right) Y\left(\vec{x}, t\right) = 0$$

In the above equation x_1, x_2 and x_3 represent the space coordinates and t , the time. It is the function $Y\left(\vec{x}, t\right)$ which represents a wave motion traveling with a velocity v. Further, since the equation is quadratic in x_i, it remains invariant under x_i changing sign i.e., $x_i \to -x_i$ which means that the solution of the equation, $Y\left(\vec{x}, t\right)$, can represent a forward moving or a backward moving wave.

Incase of water waves it is the position of the water molecules on the surface of water that executes a wave motion and is represented by function $Y\left(\vec{x}, t\right)$. Incase of electromagnetic waves like light, radio waves or microwaves it is both electric and magnetic fields respectively represented by $Y\left(\vec{x}, t\right)$ which propagate in the form of waves.

Yes, strange through it may sound, light happens to be a very special kind of combination of electric

and magnetic fields. This was illustrated by James Clerk Maxwell through his four famous equation written in 1861 when he was a young man of thirty. Experimental verification of Maxwell's assertion of light being an electromagnetic wave came through the work of Frank Hertz in 1888. Incase of sound, the displacement of individual atoms/molecules of the medium in which sound moves forms a wave motion. Thus sound wave is just a combination of compression and rarefaction of the medium. Sound waves, evidently, can not propagate in the absence of a medium whereas light waves can. In the case quantum mechanics, the function $Y\left(\vec{x}, t\right)$ essentially becomes the wave function ψ representing the amplitude of the de-Broglie probability wave associated with the microscopic quantum particle albeit a relativistic one. One mathematical equation suffices to describe wave motion in all its avataras!

Perhaps the most interesting and intriguing fact about the above equation is its relativistic or Lorentz invariance even though it was written down more than a century before Einstein created his Special Theory of Relativity. The relativistic invariance of wave equation is manifest since it contains second order derivatives of both space and time coordinates. As far as the wave equation is concerned, space and

time coordinates are treated at par. This equation has been telling us of the space-time unity all along over a century, as it were, which was concretized in the magic hands of Einstein.

The wave equation, as we can see, is both simple and beautiful indeed!

How wonderful it is that the Universe-waves are rising at once with the emergence of mind-wind in me-the limitless ocean!

-ASTAVAKRA SAMHITA

EMMY NOETHER

1882-1935

Emmy Noether was born in Erlangen, Germany on 13th March, 1882. Her father Max Noether was a distinguished mathematician. Emmy was the eldest of the four siblings of her parents and the only daughter. She was fond of dancing in her childhood.

After finishing her school studies in 1900 she obtained the certificate to teach English and French languages. But she was attracted to mathematics. On completion of her studies in Nuremburg, she went to the famous University of Göttingen to study Mathematics. In Göttingen, she had lessons of mathematics from world-famous mathematicians like Hilbert, Klein and Minkowski. She obtained her Ph.D in 1907.

She could not, however, become a teacher in the University as the prevailing rules of the University

did not permit a lady to become so. She returned to her home in Erlangen. There, in her own home, she took care of her father and also of mathematics through research. With publication of her research works, her stature grew. In 1945 Hilbert and Klein took her back to Göttingen. She started to teach there but the official allotment of the classes were in the name of Hilbert. Consequent to persistent efforts by Hilbert, Klein and others, Noether was finally appointed as a teacher in the University in 1919.

The first paper Noether wrote after joining Göttingen was on the relationship between symmetries and conservation rules published in 1918. This is famous as Noether's theorem in modern Physics. The conserved currents resulting from symmetries are also called Noether currents. The Noether's theorem has found innumerable applications in various areas of Physics including the most modern idea of unifying all four fundamental interactions using local gauge symmetries.

Emmy Noether was a famous mathematician of her time. She was awarded the prestigious Alfred Ackermann-Toibre memorial prize for her significant contribution to the advancement of mathematics.

With the rise of Nazi regime in Germanny, she not only lost her job but also had to quit Germany.

She took up a professor position in Brian-Moyer College, Pennsylvania, U.S.. She used to also teach in the famous Institute of Advanced studies, Princeton, at times.

Noether died on 14th April 1935.

EQUATION OF SYMMETRY

Da Vinci's Madonna Litta and the Child

In his book, "The Architecture of Twentieth Century" the French architect Le Corbusier writes, "Man needs order, without it all his actions lose concordance, logical interplay. The more perfect is the order, the more comfortable and confident is man. He makes mental constructs on the basis of the order that is dictated to him by the needs of his psychology. This is the creative process. Creation is an act of

ordering. " The same is true of music, literature and science-in all acts of creation, in fact.

In the process of scientific cognition of the physical world, the act of ordering is greatly facilitated by recognition of symmetry; be it the visible ones of geometrical structures or be it the much deeper ones of physical phenomena. It is the latter ones and their universality, which have become the hallmark of twentieth century scientific insight. The fundamental symmetry principles govern the laws of Nature, which, in turn, govern the physical phenomena. Symmetry underlies the theories of relativity, quantum mechanics, solid state, atomic, nuclear and particle physics and more noticeably biology.

Symmetry, from the most general point of view, implies invariance under certain transformations. A circle is symmetric with respect to rotation by any angle about its centre. A sphere enjoys similar symmetries about infinity number of axes passing through its centre. An equilateral triangle, on the other hand, enjoys symmetry under rotation about its centroid only for angles which are integral multiples of 120^0. Crystallographic structures are deciphered on the basis of such geometrical symmetries. Such symmetries conserve geometrical shapes.

Going a step deeper, there exist other conservation principles of Nature which arise out of symmetries of physical systems under transformations like translation and rotation. Space-translation invariance results in conservation of linear momentum. Time-translation invariance leads to conservation of energy. Rotational invariance engenders conservation of angular momentum. We all know that without conservation laws such as above, our world would not simply exist. They are the invisible wheels on which the entire chariot of creation quietly rolls. Incredibly enough, all these diverse conservation laws exhibit splendid unity of mathematical nature in terms of the 4-dimensional conservation equation of Noether's theorem,

$$\partial_{\mu} J_{\mu\nu\alpha...} = 0$$

Here $J_{\mu\nu\alpha...}$ is a general tensor which in appropriate situations would represent a conserved charge or energy or momentum etc. It is called a conserved "current" in recognition of pioneering work by Noether in connecting symmetry and conservation of physical observables. ∂_{μ} represent 4-dimensional derivative. The above equation reads, as "the 4-divergence of a conserved current is zero". It is now evident that invariance or symmetry under a particular transformation has an associated conserved

current (J_μ) which satisfies conservation equation given above.

Transformations need not always be defined in external space such as translation or rotation which depend on space and time coordinates. There can be transformations in, what one calls, "internal space". These do not depend on space and time coordinates. Most interesting ones of such type are basically phase transformations of the type where phase θ is a scalar function. They are also called global gauge transformation. It is simply a constant and if a physical system is invariant under such a transformation, then there exist an associated conserved current. It leads to conservation of numbers like electric charge, lepton number, baryon number, hypercharge etc. If instead, θ carries an index rendering it a vectorial character with respect to an internal space, scalar phase can be written as $\theta^a G^a$ where $G^a s$ are the generators of a group G which constitute the transformations in the internal space. $\theta^a G^a$ is basically a scalar product of two vectors of the internal group-space. If the group happens to be group of unitary unimodular transformations in 2-dimensions i.e. SU (2) we have mathematical description of spin and isospin. Neutron and proton are the basic doublets of isospin SU (2). If say, the group happens to be the group of unitary

unimodular transformation in 3-D dimensions like SU (3), we have the Gell-Mann's SU(3) classification model of elementary particles. The up, down and strange quarks are the basic triplets of this SU (3). On the basis of this classification, Gell-Mann predicted the existence of a particle Ω - which was looked for and found! This reminds us of classic predictions of a number of elements in periodic table by Mendeleev or prediction of existence and position of Neptune by U.Le. Verrier and John Adams in 1846.

Going a step still deeper, one can make θ^a dependent on space and time coordinates. One then has local gauge transformations, which too may form groups like U(1), SU(2) or SU(3) or even more complicated ones in combination. It is in this realm that the most delightful and fundamental insight of modern physics has presented itself. The various fundamental interactions and their interrelationships are understood as local gauge theories!!! The electromagnetic interaction is a U(1) gauge theory. The strong interaction is a local SU(3) gauge theory. The relationship between electromagnetic and weak interaction is established through local SU(2) $\otimes$ U(1) gauge theory. Physicists all the world over, are busy in constructing various gauge models for their dream theory of unification of all fundamental interactions.

Symmetry, thus, seems to dictate dynamics. Here, thus one finds, the notion that "symmetry authors laws of nature and laws of nature govern the physical phenomena" is operating at its flowery best.

The mathematics of symmetry is certainly as simple as profound in its physical content. Simple is beautiful, indeed!!!.

Nature-Science-art. In all of these we find the age-old competition of symmetry and asymmetry.

- L.TARASOV

Ludwig E. Boltzmann

1844-1906

Ludwig E. Boltzmann was born in Vienna on February 20, 1844. He received his primary education from a private tutor in his house. He entered the Gymnasium in Linz later and was always the best in his class. Boltzmann entered the University of Vienna in 1863 to study mathematics and physics. He was awarded the doctoral degree in 1866.

He occupied the professor positions in the Universities of Graz and Vienna of Austria, Munich and Leipzig of Germany. He was not only a brilliant physicist but his thoughts spreaded to life sciences and philosophy, most of which are contained in his brilliant book, "Popular Writings".

He received piano lessons from Anton Brückner and used to play chamber music through out of his life.

While still fighting for global acceptance of his atomistic view, he committed suicide on 5 September 1906 during a vacation in Duino in Italy.

His famous formula S=K log W, though actually written in this form by Max Planck, is engraved on his tombstone in central cemetery in Vienna, situated next to the graves of the presidents of the Austrian republic.

BOLTZMANN'S STATISTICS

Man loves order. It is order that brings out a great piece of prose out of chaos of events and experiences, it is order that creates poetry out of the chaos of words, it is order of thoughts that builds a philosophy, it is order that fashions a personality out of the chaos of habits, it is the order that provides a structure to the Universe out of the chaos of possibilities. Order does not just appeal to emotion and intuition in an abstract manner only, it can also be quantified at least for the physical systems ala' the physics way.

The attribute of a physical system that relates to disorder is called entropy. Entropy is a measure of the disorder in the system; greater the disorder greater is the entropy. This connection between entropy and disorder was first shown by Ludwig Boltzmann in the year 1877. Thus the idea of entropy, first introduced by Rudolf Clausius in 1857 as an abstract mathematical concept, was related to familiar facts of everyday life by Boltzmann. He even gave the mathematical formula that relates entropy and

disorder. Planck was the first man to put this relationship in mathematically neat form as

$$S=K \log W$$

Here W is the probability to find a physical system in a class of states. This equation and constant K are famous today as Boltzmann's equation and Boltzmann's constant respectively. This equation, as a fitting tribute to its inventor's insight, is engraved on Boltzmann's tombstone in the central cemetery of Vienna, situated next to the graves of the presidents of Austrian republic.

The most important implication of the Boltzmann's equation is the statistical interpretation of the basic laws of classical physics like the laws of thermodynamics. In small space regions of a gas, about 10^{-3} cm in size, deviations from laws of thermodynamics known as statistical fluctuations do exist. But in the thermodynamic limit, one can predict the general properties of a very large- ideally infinitely large-material system. The laws one obtains in such a limit are just the laws of thermodynamics. Statistical physics, thus, allow us to make fairly definitive predictions of physical properties of a system, with large number of constituents. The larger the number, better is the prediction!

Coming back to the equation, let us clarify the notion of "probability of finding the system in a class of states". Let us imagine a vessel divided into two equal parts (left and right) containing only 4 molecules. Each of the molecules may be found in the left half or right with equal probability. This system has 5 possible macrostates: 1-there are no molecule in the left half, 2-there is 1 molecule in the left half, 3-there are two molecules in the left half 4-3 molecules in the left half and 5-there are all 4 molecules in the left half. These macro probable states are realized by different numbers of equally probable ways. The macrostates-1 and 5 only occur in one way each and thus correspond to one microstate. Macrostates-2 and 4 correspond to four microstates since any one of four molecules may stay in left or right half. Macrostate-3 corresponds to six equally probable microstates. Thus, there are sixteen equally probable microstates in all. The probability of a macro state is proportional to the number of corresponding microstates. This is the probability involved in Boltzmann's formula. In this example probabilities associated with each of the macrostates 1 and 5 is 1/16, that of the macrostates 2 and 4 is ¼ and of the microstate 3 is 3/8.

Further, we see that macrostate-1 and 5 exhibit a structure of the system with all molecules in one half

and no molecule in the other. But the macrostate 3 has no distinguishable structure with equal molecules in both the halves. Since presence of a structure indicates order in a system and its absence disorder, the probability of a macrostate gets related to disorder. The greater the probability, greater the disorder and hence greater is the entropy.

How beautifully the underlying statistical nature of a classical system is brought out by the simple Boltzmann's equation !

Simple, once again, is found to be beautiful too.

Even as a musician can recognize his Mozart, Beethoven or Schubert after hearing the first few bars, so can a mathematician recognize his Cauchy, Gauss, Jacobi, Helmoltz or Kirchoff after first few pages.

BOLTZMANN

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Control by experiments is, ofcourse, an essential ingredient prerequisite of the validity of any theory. But one can not possibly test everything. That was why I am so intrested about simplicity. Still, I should never claim that I really understand what is meant by simplicity of natural laws.

- Einstein

L.P. Singh was a professor of physics at the Utkal University, Bhubaneswar, India. He was the recipient of best graduate Mayurbhanj gold medal of Utkal University in the year 1967. He had his Post-M.Sc. associateship from Saha Institute of Nuclear Physics, Kolkata, Ph.D. from Utkal University and research stints at Dortmund University, Germany; Dublin Institute for Advanced Studies, Dublin; Abdus Salam International Centre for theoretical Physics, Trieste and CERN, Geneva. He is the author of 'Anuru Antarikhsa' which won Orissa Bigyan Academy Award as the best popular science writing in 1999.

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